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This project was concerned with problems in operator theory and matrix theory that underlie the maximum entropy principle in signal processing and systems theory. Two papers have been completed that generalize this principle to a wide class of indefinite hermitian matrices.		signal processing, control theory, autocorrelation matrix, Toeplitz matrix, rank-preserving extensions, linear systems, partial realizations, interpolation problems, sensitivity minimization, Gohberg-Semencul formula, orthogonal polynomials, matrix theory.		
Three new papers give a thorough study of rank-preserving extensions of band matrices. Factorization theorems are obtained for wide classes of Toeplitz and Hankel matrices, and connections are given to the Kalman partial realization problems in signal processing. Applications are also given to signal processing when the power spectrum consists of a set of pure line spectra. In another paper, new work on orthogonal polynomials and the Gohberg-Semencul formula for the inverse of a Toeplitz matrix has been completed.				
Three new papers generalize for rational matrix functions a number of well-known interpolation problems for scalar functions. A new approach to tangential interpolation problems is presented, and applications are given to sensitivity minimization in control theory.				
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APPLICATIONS OF OPERATOR THEORY TO MAXIMUM ENTROPY PROBLEMS

Robert L. Ellis, Israel Gohberg, and David C. Lay

FINAL REPORT

Summary This project was concerned with problems in operator theory and matrix theory that underlie the maximum entropy principle in signal processing and systems theory.

- ◆ Two papers have been completed that generalize the maximum entropy principle to a wide class of indefinite hermitian matrices.

- ◆ New work on orthogonal polynomials and the Gohberg-Semencul formula for the inverse of a Toeplitz matrix has been completed, and this work has stimulated considerable research activity by other authors.

- ◆ Three papers have been completed that give a thorough study of rank-preserving extensions of band matrices. Factorization theorems are obtained for wide classes of Toeplitz matrices and Hankel matrices, and connections are given to the Kalman partial realization problem in signal processing. Applications are also given to signal processing when the power spectrum consists of a set of pure line spectra.

- ◆ Three papers have been completed that generalize for rational matrix functions a number of well-known interpolation problems for scalar functions, and a new approach to tangential interpolation problems is presented. Applications are given to sensitivity minimization in control theory.

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Applications of Operator Theory to Maximum Entropy Problems

Robert L. Ellis, Israel Gohberg, and David C. Lay

The use of the maximum entropy principle (MEP) or method (MEM) is well known in the study of stationary stochastic processes. Simply stated, the MEP is used to construct a covariance function of a process in a situation where the data provides only an estimate for "part" of the function. The technique of constructing or "extending" a function having specified properties has been studied in a variety of cases by many mathematicians, beginning with Caratheodory and Toeplitz in the early 1900's. In the 1970's, Burg [4] introduced the MEP which has been so influential in signal processing. We have analyzed various aspects of this problem in 5 publications (and one of our group, I. Gohberg, has coauthored an additional 19 papers on the subject over the past ten years). Until recently, research has focused on either (1) functions that are positive and matrices (possibly infinite) that are positive definite, or (2) functions of modulus less than one and matrices (or operators) that are contractions. Problems in these two areas are intimately connected, and solutions in one area often may be translated into solutions in the other area.

In 1986, a new maximum entropy principle for contractions was published in [5]. The importance of this principle for H^∞ -control problems was discovered recently by the research engineers Keith Glover and John Doyle, who described in [13] the significance of maximum entropy in the problem of risk sensitive log control. However, they also showed that a MEP is needed for extension problems more general than those described in (1) and (2) above.

The first successful attempt to broaden the MEP in a finite-dimensional setting to indefinite hermitian matrices was done by our research group [6]. The work continued in [7,8], supported by the AFOSR. We found a MEP for certain broad classes of hermitian band matrices, and we examined some cases where a MEP exists only as a

constrained maximum. Our work explains why a MEP cannot exist for all hermitian matrices: in general the band extension of a hermitian matrix is only a stationary point for the determinant function and can yield a maximum, a minimum, or a saddle point. For Toeplitz matrices, however, the general selfadjoint case is fairly close to the positive definite case that arises in signal processing and other situations involving stationary stochastic processes.

When the power spectrum of a signal consists of a set of pure line spectra, the autocorrelation matrix $T(n)$ is singular for n larger than some N . We have found that the higher order Fourier coefficients of the spectrum are completely determined by the requirement that the associated extensions of $T(N)$ have minimum rank, and we have an explicit formula for these matrix extensions. Our results stem from a thorough study of rank-preserving extensions of band matrices [10,11]. Applications of this work are given to the problem of extending a triangular matrix to a unitary matrix or a contraction.

A factorization formula in [10] for a positive semidefinite Toeplitz matrix has been generalized in [12] to a wider class of Toeplitz matrices, and an analogous formula for Hankel matrices has been discovered. As mentioned in [12], the Hankel case is connected to the Kalman partial realization problem in signal processing,

In another direction, our work [9] on polynomials orthogonal on the unit circle has turned out to be of wide interest. We gave a new proof of Krein's theorem on the distribution of the zeros of orthogonal polynomials, a result that generalizes the standard minimum phase theorem for error prediction filters. We showed the connection between Krein's theorem and the Gohberg-Semencul formula for the inverse of a Toeplitz matrix. This work has stimulated at least four other papers on generalizations and related topics (see [14]).

As we pointed out in our proposal, some of the problems we are studying may be approached as interpolation problems. In the past two years, substantial progress has been made in this area by Gohberg, together with Ball and Rodman [1,2,3]. They



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have developed a new approach to tangential interpolation problems, including the problem of reconstructing a rational transfer function from its null and pole data, and the problem of constructing a unitary rational function when only a part of the null and pole data are given. Problems of Nevanlinna-Pick type are also considered. The solutions of these problems are given in an explicit form, within the framework of realization theory and finite-dimensional matrix analysis. Applications are given in [3] to sensitivity minimization associated with stabilizing compensators for a regular plant, an important area of interest in system theory.

Honors and Recognitions In August, 1988, an international conference, attended by nearly 100 mathematicians and engineers, was held at the University of Calgary in honor of Israel Gohberg's 60th birthday. Ellis and Lay were among the 27 invited speakers at the week-long conference. All three investigators have been invited speakers each year at national or international conferences.

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